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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

275. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Given the simultaneous equations $x^y-y^x=0$ and $y-x=a(a+1)^{1/a}$; find a solution which is real when $a>-1$.

Solution by A. H. HOLMES, Brunswick, Maine, and L. E. NEWCOMB, Los Gatos, California.

$x^y=y^x$...(1); $y-x=a(a+1)^{1/a}$...(2). Put $y=x^z$. Then in (1) $x^{x^z}=x^{zx}$. $\therefore x^z=zx$. From (2), $x(x^{z-1}-1)=a(a+1)^{1/a}$, or $x(z-1)=a(a+1)^{1/a}$. Take $(z-1)=a$. Then $x=(a+1)^{1/a}$, $z=a+1$, $y=zx=(a+1)^{(a+1)/a}$. Putting for y and x these values in (1),

$$(a+1)^{(1/a)\,(a+1)^{(a+1)/a}}=(a+1)^{[(a+1)/a](a+1)^{1/a}};$$

$$\frac{(a+1)^{a+1}}{a}\log(a+1)=\frac{a+1}{a}\times(a+1)^{1/a}\log(a+1);\text{ and}$$

$$\frac{a+1}{a}=\frac{a+1}{a}.$$

$\therefore x=(a+1)^{1/a}$ and $y=(a+1)^{(a+1)/a}$, which are real when $a>-1$.

Also solved by G. B. M. Zerr and J. Scheffer.

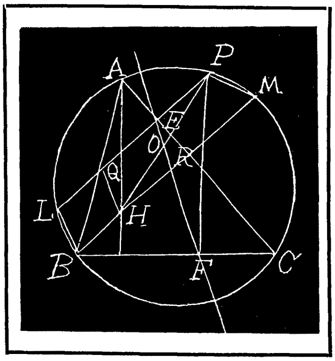
No solution of 276 has yet been received.

GEOMETRY.

303. Proposed by FRANCIS RUST, C. E., Allegheny, Pa.

Prove that the pedal line of any point on a triangle's circum-circle bisects the distance from this point to the triangle's ortho-center.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va., and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.



Let H be the ortho-center, P the given point, EF the pedal line intersecting PH in O. Produce the perpendicular BR to meet the circumference in M, and produce PE to meet the circumference in L, also take $EQ=PE$.

Quadrilateral $PEFC$ is inscriptible; $\angle PCF$ =supplement of angle PLB, both measured by $\frac{1}{2}$ arc PAB.

$\therefore \angle PEF$, the supplement of $\angle PCF=\angle PLB$. $\therefore EF$ is parallel to LB.

Now $RM=RH$, $PD=EQ$, hence the trapezoid $PQHM$ is isosceles. $\therefore PM=QH$, but $PM=LB$. $\therefore LB=QH$ and is parallel to it since $\angle PLB=\angle PQH=\angle PEF$. $\therefore EF$ is parallel to QH.

Now E is the midpoint of PQ, hence O is the midpoint of PH.

Also solved by the Proposer.

304. Proposed by G. W. GREENWOOD, M. A., Dunbar, Pa.

Find the tangent at the points $(a, 0)$ and $(0, a)$ to the locus $x^3+y^3=a^3$, and show that these points are points of inflexion.

I. Solution by A. H. HOLMES, Brunswick, Maine.

$x^3+y^3=a^3$. $\therefore \dfrac{dy}{dx}=-\dfrac{x^2}{(a^3-x^3)^{2/3}}$, which is 0 for $x=0$, and ∞ for $x=a$.

$\dfrac{d^2y}{dx^2}=-\dfrac{2x(a^3-x^3)^{1/3}-2x^4}{a^3-x^3}$, which is 0 for $x=0$, and ∞ for $x=a$. Take $x>a$,

and $\dfrac{d^2y}{dx^2}$ is seen to be minus. Take $x<a$ (a little) and $\dfrac{d^2y}{dx^2}$ is plus.

$\therefore (a, 0)$ and $(0, a)$ are points of inflexion.

II. Solution by BENJ. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

We have for the slope of the curve at any point, $\dfrac{dy}{dx}=-\dfrac{x^2}{y^2}$. $\therefore y-y_1=-\dfrac{x_1^2}{y_1^2}(x-x_1)$ is the equation of the tangent at any arbitrary point (x_1, y_1) of the curve. For $(0, a)$, the equation of the tangent is $y-a=0$. For $(a, 0)$, the equation of the tangent is $x-a=0$. From the equation of the tangent we have $y=y_1-\dfrac{x_1^2}{y_1^2}(x-x_1)$. In this equation, find y, for $x=x_1-h$ and $x=x_1+h$; also find the corresponding values of y from the equation of the curve, $y=\sqrt[3]{(a^3-x^3)}$. If the differences of these corresponding values of y change signs, the point is a point of inflection; if they do not, the point is an ordinary point of tangency. From the equation of the tangent, the values of y for the point $(0, a)$ are $y'=a$, $y''=a$, and from the curve ${}'y=\sqrt[3]{(a^3+h^3)}$, ${}''y=\sqrt[3]{(a^3-h^3)}$; ${}'y-y'=\sqrt[3]{(a^3+h^3)}-a>0$, ${}'y-y'=\sqrt[3]{(a^3-h^3)}-a<0$. Hence, $(0, a)$ is a point of inflection.

Similarly for the point $(a, 0)$, $y'=\infty$, $y''=-\infty$.
${}'y=\sqrt[3]{(3a^2h-3ah^2+h^3)}$, ${}''y=\sqrt[3]{(-3a^2h-3ah^2-h^3)}$,
${}'y-y'=\sqrt[3]{(3a^2h-3ah^2+h^3)}-\infty<0$, ${}''y-y''=\sqrt[3]{(-3a^2h-3ah^2-h^3)}+\infty>0$.

Hence, the point $(a, 0)$ is a point of inflection.

Also solved by G. B. M. Zerr, J. Scheffer, and the Proposer.